From the nonlinear dynamical systems theory
to observational chaos workshop
Toulouse, France

Multistability in the spin-orbit dynamics of celestial bodies

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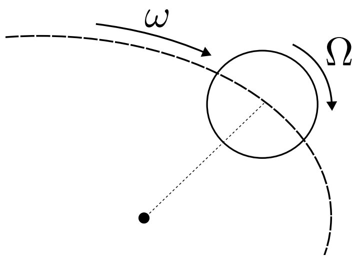
Introduction



Spin-orbit resonance (SOR)

$$p/q = \Omega/\omega$$

Examples in the Solar System

- Earth's Moon
  - 1/1 resonance
- Pluto and Charon
  - 1/1 resonance (both)
- Many other moons (Phobos, Deimos, Io, Europa, Ganymede, Callisto, ...)
  - 1/1 resonance

The case of Mercury

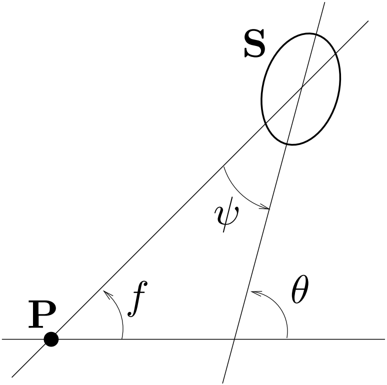
- 3/2 spin-orbit resonance
  - Why? First thought to be in a 1/1 SOR

- Some works on the problem
  - Pettengill & Dyce (1965) $\Rightarrow$ Rotation period of Mercury determined by radar
  - Goldreich & Peale (1966, 1970) $\Rightarrow$ Derivation of a capture probability formula based on energy arguments
  - Henrard (1985) $\Rightarrow$ Probability of capture reinterpreted in terms of adiabatic invariant theory
  - Correia & Laskar (2004) $\Rightarrow$ Chaotic evolution of Mercury's orbit can drive its eccentricity very high during the planet's history, leading to a higher capture probability
  - Celletti & Chierchia (2008) $\Rightarrow$ Basins of attraction of higher-order SOR are bigger for higher eccentricities

In this work

- Our aim here is to investigate the mechanism leading to the capture into spin-orbit resonances
- We do that by visually depicting and also quantifying, via the Gibbs entropy, the complexity of the basins of attraction in the system
- Our results highlight the rich dynamical scenarios that may emerge from such a system

Physical model



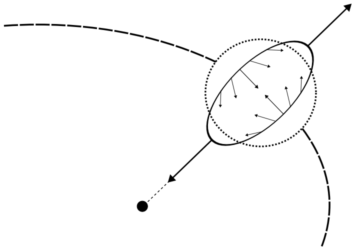
$\mathbf{S}$: Satellite or Planet (triaxial)
$\mathbf{P}$: Planet or Star (point mass)
f: true anomaly
θ: rotation angle

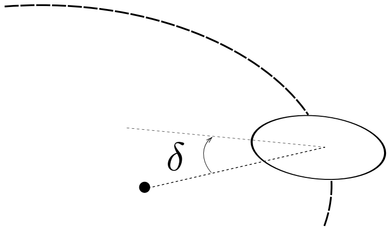
Assumptions

- $\mathbf{S}$ orbits $\mathbf{P}$ in a fixed Keplerian ellipse with semi-major axis a, eccentricity e, and instantaneous radius r
- spin axis parallel to the largest principal moment of inertia and perpendicular to the orbit plane
- the only forces that act on $\mathbf{S}$ are the ones generated by the gravitational field of $\mathbf{P}$

Tidal Forces

- Caused by the gradient formed by the central body's gravitational field, which stretches the orbiting body
- Main mechanism for dissipating energy and trapping orbiting bodies into spin-orbit resonances
- Higher effect on **larger** and **closer** bodies
- There are different tidal models. Here, the time lag between the body's distortion and the tide-raising potential is assumed constant





Spin dynamics of an almost rigid body

Equations of motion

$$\ddot{\theta} = -\gamma \frac{G m_P}{r^3} \sin 2(\theta - f) - K \left[L(r)\dot{\theta} - N(r, e) \right]$$

Physically relevant parameters

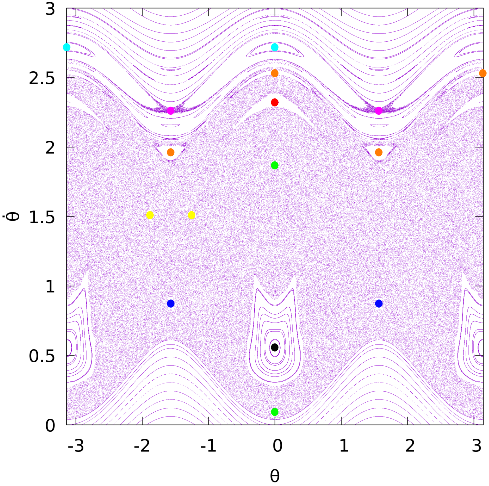
orbital eccentricity	equatorial oblateness	dissipation constant
e	$\gamma := \frac{3}{2}\frac{I_2 - I_1}{I_3}$	K

$\Rightarrow$ In reality, the parameters of the problem are not constant in time

Hyperion

- Moon of Saturn
- Chaotic rotation
- Very aspherical shape, being nearly twice as long as it is across (Voyager 2)
- Physical parameters: $e \approx 0.1$ and $\gamma \approx 0.264$
- Very nice test case (even though our model does not apply)
- Main references: Wisdom and Peale (1984) and Wisdom (1987)

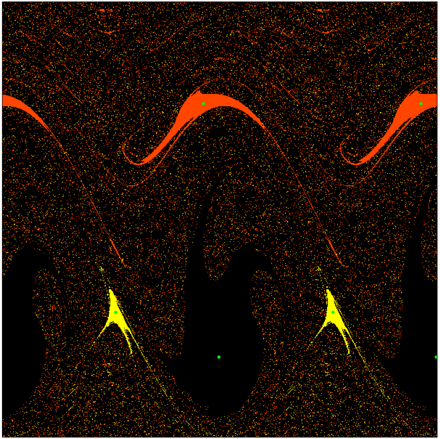
Phase space – $\theta \times \dot{\theta}$



Resonances:

- 1/1 stable – black
- 2/1 stable – red
- 1/2 stable – blue
- 2/2 unstable – green
- 3/2 unstable – yellow
- 5/2 stable – light blue
- 5/2 unstable – pink
- 9/4 stable – orange

Basins of attraction for Hyperion ($K = 10^{-2}$)

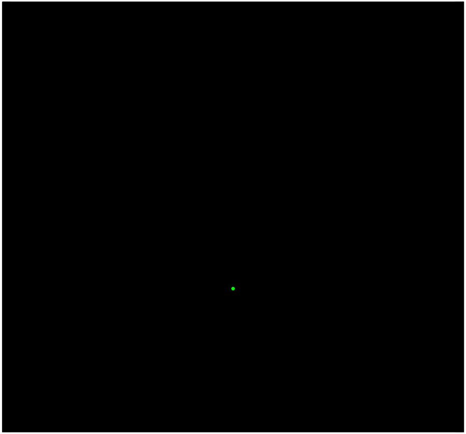


- 1/1 – black
- 2/1 – orange
- 1/2 – yellow

Basin of attraction: set of trajectories that converge in time to a given attractor.

Basins of attraction varying e for $\gamma = \gamma_{hyp}$ (gif)

e = 0.00



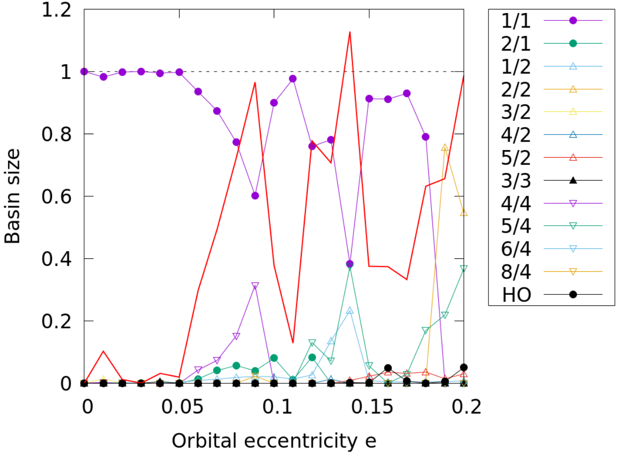
Entropy

Definition

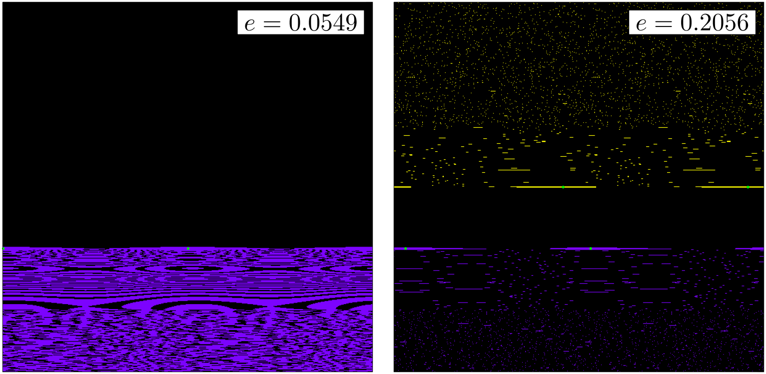
$$S = \sum_{i=1}^{N_A} p_i \ln\left(\frac{1}{p_i}\right)$$

- N_A is the number of attractors, which depends on the system parameters
- p_i is the probability of an orbit belonging to the basin of attraction of the i-th attractor, which corresponds to its basin size
- S is maximum when all basins have the same size, and its value is given by $S_{max} = \ln N_A$
- The entropy S reflects how heterogeneous the basin sizes are, highlighting the presence of dominant attractors

Basin sizes and entropy as a function of e for $\gamma = \gamma_{hyp}$

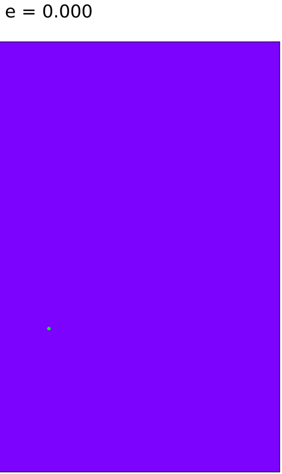


Basins of attraction for the Moon & Mercury ($K = 10^{-4}$)

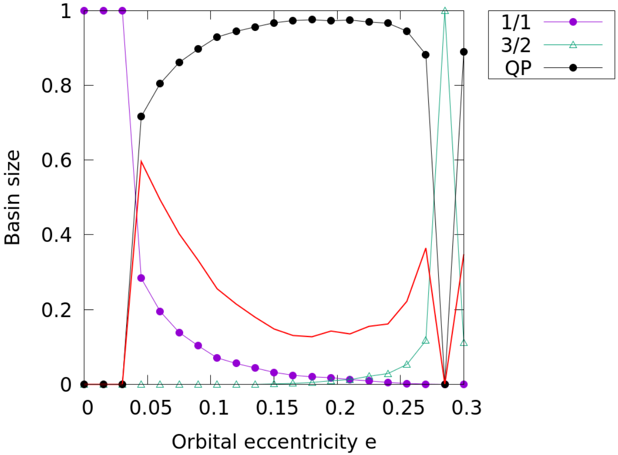


- $\gamma = 10^{-4}$ for both cases
- The color code corresponds to the following attractors: quasi-periodic – black, 1/1 – purple, and 3/2 – yellow

Basins of attraction varying e for $\gamma = 10^{-4}$ (gif)



Basin sizes and entropy as a function of e for $\gamma = 10^{-4}$



Conclusions

General

- Existence of multistability in the problem with basins of attraction showing a very intricate structure

Hyperion

- There are no quasi-periodic attractors
- Entropy tendency follows the $1/1$ SOR, which eventually bifurcates to a $2/2$ SOR

Moon & Mercury

- The basin of quasi-periodic attractors act as a barrier between the basins of the synchronous $1/1$ SOR and the $3/2$ SOR
- The basin of quasi-periodic attractors dominates the phase space when it exists

Entropy

- Easily extended to higher-dimensional models
- Good convergence criterion for Monte-Carlo simulations

Future & more information

- Use a more realistic rheological model
- Study the effect of long term variations of spin and tidal forces

- Article's preprint titled "Multistability and Gibbs entropy in the planar dissipative spin-orbit problem" is available at ArXiv
  - `arxiv.org/abs/2307.12969`
- The open-source software developed for this work is available at the author's Github page
  - `github.com/vitor-de-oliveira/spin-orbit`

Thank you!

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