Using Auxiliary Information in Model Building for Nonlinear Dynamics

An Application in Robotics

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Introduction

Overview

What is system identification?
How is it accomplished?

- Testing and data collection
- Choice of model class
- Structure selection
- Parameter estimation
- Model validation

Introduction

Black-Box Identification

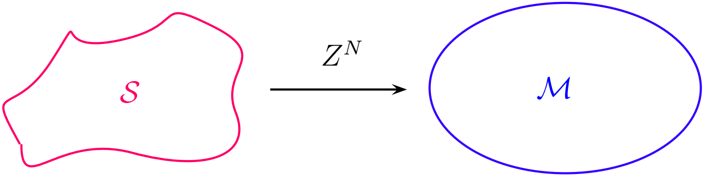


Figure: Simplified schematic diagram for *black-box identification*, where $\mathcal{S}$ represents the system that should be approximated by a model $\mathcal{M}$ which is built from a set of measured data Z^N of length N.

Introduction

Grey-Box Identification

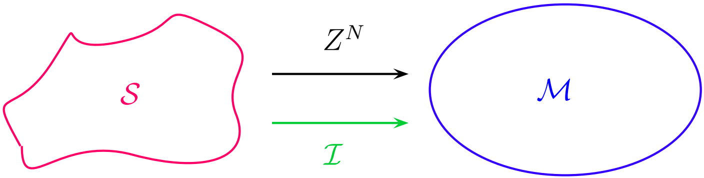


Figure: Simplified schematic diagram for *grey-box identification*, where $\mathcal{S}$ represents the system that should be approximated by a model $\mathcal{M}$ which is built from a set of measured data Z^N of length N *and* auxiliary information $\mathcal{I}$.

Introduction

Grey-Box Identification: Questions

1. What kind of auxiliary information $\mathcal{I}$ is useful?
2. How does $\mathcal{I}$ relate to the model class?
3. Assuming that $\mathcal{I}$ is compatible with the model class, how do we actually use $\mathcal{I}$ in determining the final model?

Using Auxiliary Information

Types of Auxiliary Information

For linear systems:

1. DC gain;
2. stability.

For nonlinear systems:

1. static function (calibration curve);
2. steady-state data;
3. number, position and symmetry of fixed points;
4. fixed point bifurcations;
5. hysteresis.

Using Auxiliary Information

Steady-state data: An example

Given the model structure:

$$
\begin{aligned}
y(k) \; = \; & \theta_1 y(k-1) + \theta_2 y(k-2) + \theta_3 u(k-1) + \theta_4 u(k-2)^2 \\
& + \theta_5 u(k-1)u(k-2) + \theta_6 u(k-2)
\end{aligned}
$$

and the data sets $Z^N = [u(k), y(k)],\ k = 1, 2, \ldots, N$ and $\mathcal{I}:\ Z_{\rm ss}^M = [\bar{\rm u}, \bar{\rm y}]$, where $\bar{\rm u} = [\bar{u}_1, \ldots, \bar{u}_M]^{\rm T}$ and $\bar{\rm y} = [\bar{y}_1, \ldots, \bar{y}_M]^{\rm T}$. The aim is to estimate the parameters from Z^N such that the final model has $Z_{\rm ss}$, say $[\bar{u}_3, \bar{y}_3]$ and $[\bar{u}_7, \bar{y}_7]$ as steady-state solutions.

In order to use a Constrained Least Squares Algorithm, the constraints must be represented in the form: $S\hat{\boldsymbol{\theta}} = {\rm c}$.

Using Auxiliary Information

Steady-state data: An example

The model in steady-state yields:

$$\bar{y} = (\theta_1 + \theta_2)\bar{y} + (\theta_3 + \theta_6)\bar{u} + (\theta_4 + \theta_5)\bar{u}^2.$$

Hence the two constraints are

$$\begin{aligned}
\bar{y}_3 &= (\theta_1 + \theta_2)\bar{y}_3 + (\theta_3 + \theta_6)\bar{u}_3 + (\theta_4 + \theta_5)\bar{u}_3^2 \\
\bar{y}_7 &= (\theta_1 + \theta_2)\bar{y}_7 + (\theta_3 + \theta_6)\bar{u}_7 + (\theta_4 + \theta_5)\bar{u}_7^2,
\end{aligned}$$

which can be rewritten as $\mathrm{c} = S\boldsymbol{\theta}$ with

$$\mathrm{c} = \begin{bmatrix} \bar{y}_3 \\ \bar{y}_7 \end{bmatrix}; \quad S = \begin{bmatrix} \bar{y}_3 & \bar{y}_3 & \bar{u}_3 & \bar{u}_3^2 & \bar{u}_3^2 & \bar{u}_3 \\ \bar{y}_7 & \bar{y}_7 & \bar{u}_7 & \bar{u}_7^2 & \bar{u}_7^2 & \bar{u}_7 \end{bmatrix}.$$

Teaching Robots

The problem

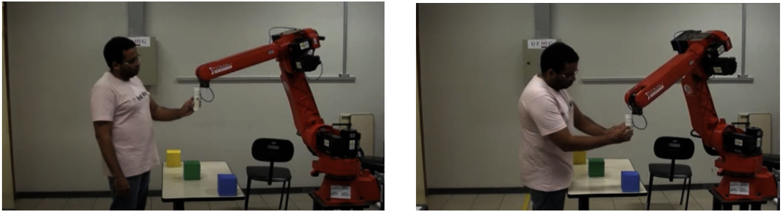


Figure: Teaching a robot by demonstration.

Teaching Robots

The challenge

- To program a robot to reach a certain location;
- Provide a trajectory to be followed;
- How to define the trajectory?
- Instead of providing a trajectory what if we provide a vector field for which infinite trajectories can be obtained, one for each possible initial condition?
- Then, a vector field becomes a trajectory-producing mechanism.

Learning reaching motions by demonstrations

Co-workers

Rafael Santos (UNIFEI, Itabira)

Guilherme Pereira (UFMG, West Virginia University)

Learning reaching motions by demonstrations

Aim

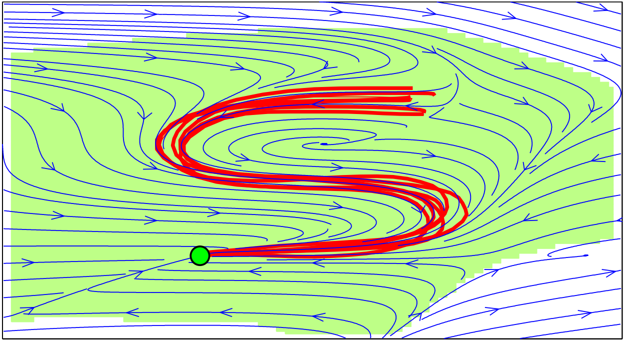
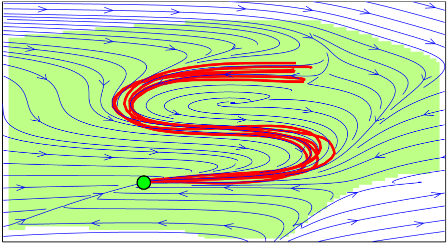


Figure: (Blue) vector field; (red) set of possible trajectories; (green shade) basin of attraction of the target, indicated by the green circle.

Learning reaching motions by demonstrations

Requirements

- The models must be autonomous;
- The target must be a stable fixed point;
- The basin of attraction should cover all demonstration data;
- The distance between demonstration data and the boundary of the basin of attraction should be the largest possible.



Learning reaching motions by demonstrations

The teacher (demonstrations)

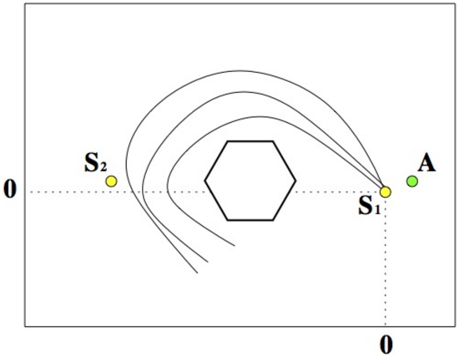


Figure: Three trajectories provided by the teacher. About 10% of the cases for black-box techniques.

Using Auxiliary Information

A simple example

$$y(k) = \theta_1 y(k-1) + \theta_2 y(k-2) + \theta_3 y(k-1)^2 + \theta_4 y(k-2)^2$$

The model in steady-state yields:

$$\bar{y} = (\theta_1 + \theta_2)\bar{y} + (\theta_3 + \theta_4)\bar{y}^2.$$

Because the model does not have any constant terms $\bar{y} = 0$ is a fixed point. The other one can be found solving

$$\begin{aligned} y^* &= (\theta_1 + \theta_2)y^* + (\theta_3 + \theta_4){y^*}^2 \\ y^* &= [y^* \; y^* \; {y^*}^2 \; {y^*}^2] \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} \end{aligned}$$

Learning reaching motions by demonstrations

The methodology

A typical 2D model has the general form:

$$\begin{aligned} y_1(k) &= F_1^{\ell}[y_1(k-1),\, y_2(k-1)] + e_1(k) \\ y_2(k) &= F_2^{\ell}[y_1(k-1),\, y_2(k-1)] + e_2(k) \end{aligned}$$

The fixed points are given by $(\bar{y}_1, \bar{y}_2)$ that are the solutions to the set of equations:

$$\begin{aligned} \bar{y}_1 &= F_1^{\ell}[\bar{y}_1, \bar{y}_2] \\ \bar{y}_2 &= F_2^{\ell}[\bar{y}_1, \bar{y}_2]. \end{aligned}$$

The stability can be established using (at such fixed points):

$$\mathrm{D}F(\boldsymbol{y}) = \left[\begin{array}{cc} \dfrac{\partial F_1^{\ell}}{\partial y_1(k-1)} & \dfrac{\partial F_1^{\ell}}{\partial y_2(k-1)} \\ \dfrac{\partial F_2^{\ell}}{\partial y_1(k-1)} & \dfrac{\partial F_2^{\ell}}{\partial y_2(k-1)} \end{array} \right].$$

Learning reaching motions by demonstrations

The methodology

$\mathcal{M}$ an unconstrained model with fixed points at $\bar{\boldsymbol{y}} = 0$ and $\bar{\boldsymbol{y}}_1^*$.
$\mathcal{M}_{\rm c}$ a constrained model with same structure, hence with $\bar{\boldsymbol{y}} = 0$, and estimated from the same data but constrained to have a new fixed point at $\bar{\boldsymbol{y}}^* = [\bar{y}_1, \bar{y}_2]^{\rm T}$:

$$\mathsf{c} = \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix}, \qquad S = \begin{bmatrix} F_1^{\ell}[\bar{y}_1, \bar{y}_2] \\ F_2^{\ell}[\bar{y}_1, \bar{y}_2] \end{bmatrix}.$$

Conjecture: $\bar{\boldsymbol{y}}^*$ of $\mathcal{M}_{\rm c}$ will be of the same type as that of $\bar{\boldsymbol{y}}_1^*$ of $\mathcal{M}$ for sufficiently small $|\bar{\boldsymbol{y}}_1^* - \bar{\boldsymbol{y}}^*|$.

Learning reaching motions by demonstrations

The methodology

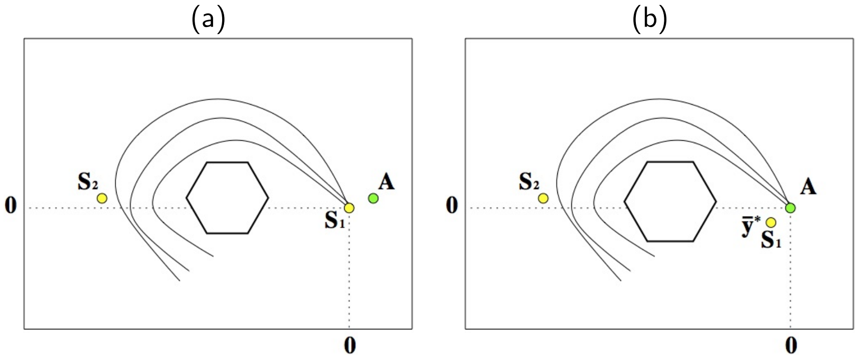


Figure: Scenarios for (a) unconstrained and (b) constrained models.

Learning reaching motions by demonstrations

An example: the data

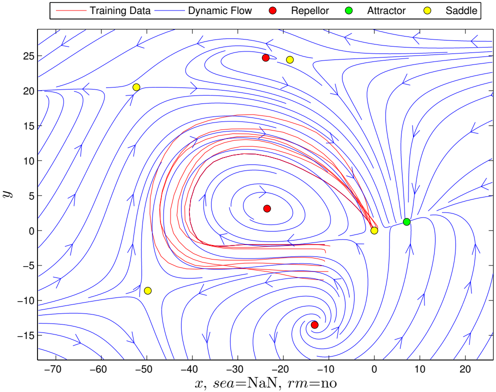
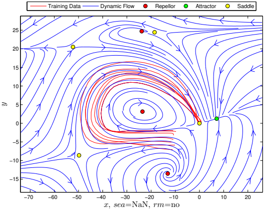


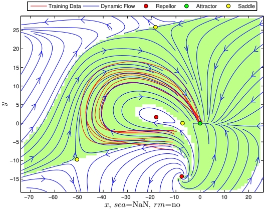
Figure: (Red) teacher-produced demonstrations; (blue) the vector field of a black-box model.

Learning reaching motions by demonstrations

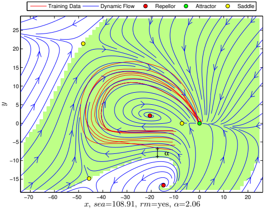
An example: a grey-box model



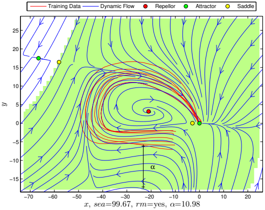
(a) Model obtained with LS estimator



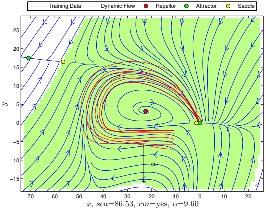
(b) After `getTargetAttractor` function.



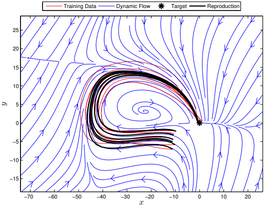
(c) After `getRM_Model` (1 and 2) functions



(d) After `optimizeAlphaDistance` function.



(e) After `optimizeAccuracy` function.



(f) Execution of the RM model.

Learning reaching motions by demonstrations

An example: a grey-box model

$$
\begin{aligned}
y_1(k) = \;& +0.983506\, y_1(k-1) + 0.096590\, y_2(k-1) \\
& -0.000078\, y_1(k-1)^3 + 0.005253\, y_2(k-1)^2 \\
& -0.000538\, y_2(k-1)^3 - 0.016513\, y_1(k-1)\; y_2(k-1) \\
& -0.000300\, y_1(k-1)^2\; y_2(k-1) - 0.004126\, y_1(k-1)^2
\end{aligned}
$$

$$
\begin{aligned}
y_2(k) = \;& +0.779775\, y_2(k-1) - 0.000042\, y_1(k-1)^3 \\
& -0.015285\, y_1(k-1)\; y_2(k-1) - 0.002493\, y_2(k-1)^2 \\
& -0.000216\, y_1(k-1)^2\; y_2(k-1) - 0.004130\, y_1(k-1) \\
& -0.000102\, y_2(k-1)^3 - 0.001130\, y_1(k-1)^2 \\
& +0.000001\, y_1(k-1)\; y_2(k-1)^2 \,.
\end{aligned}
$$

Learning reaching motions by demonstrations

The data

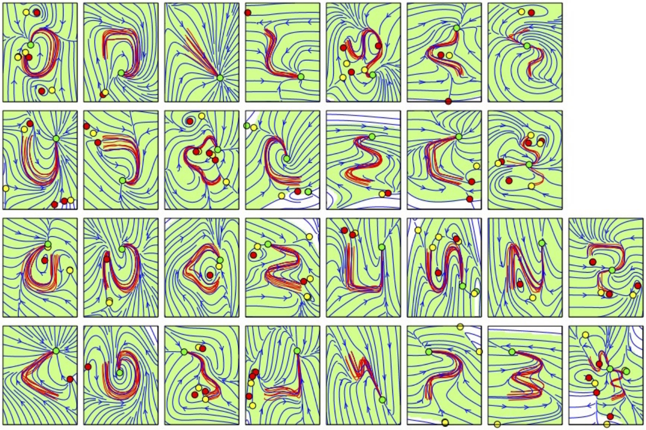


Figure: The benchmark is available at: https://www.amarsi-project.eu/benchmark-framework.

Conclusions

Other types of auxiliary information

In some applications the use of auxiliary information could be key.

- Location and symmetry of fixed points (static curve)
- Symmetry of the flow
- Bifurcations (Hopf, flip and transcritical)
- Hysteresis (multistability)
- Polynomials, MLP and RBF networks
- Constrained and multi-objective optimization

Written Material

A Bird's Eye View of Nonlinear System Identification

ResearchGate: https://www.researchgate.net/
https://arxiv.org/abs/1907.06803

Learning robot reaching motions by demonstration using nonlinear autoregressive models

Santos, R. F., Pereira, G. A. S., Aguirre, L. A. *Robotics and Autonomous Systems*, 107:182–195, 2018. DOI: 10.1016/j.robot.2018.06.006.

The videos are on YouTube

https: //goo.gl/AqMLAH.

Acknowledging your attention and patience

Thank you!

To the organizers, to the audience and to Otto Rössler: